Integral Calculus - 2

Miscellaneous Exercise Question Bank | Level - 1

1. $\int_{-1/2}^{1/2} \sqrt{\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2}\, dx$, equal to:

(A) $4\ell n\left(\frac{2}{3}\right)$ **(B)** $2\ell n\left(\frac{2}{3}\right)$ **(C)** $\ell n\left(\frac{4}{3}\right)$ **(D)** $4\ell n\left(\frac{4}{3}\right)$

2. Suppose for every integer n, $\int_{n}^{n+1} f(x)dx = n^2$. The value of $\int_{-2}^{4} f(x)dx$ is :

(A) 16 **(B)** 14 **(C)** 19 **(D)** None of these

3. $f(x)$ is a continuous function for all real values of x and satisfies $\int_0^x f(t)dt = \int_0^x t^2 f(t)dt + \frac{x^{16}}{8} + \frac{x^6}{3} + a$, then the value of '$a$' is equal to :

(A) $-\frac{1}{24}$ **(B)** $\frac{18}{168}$ **(C)** $\frac{1}{7}$ **(D)** None of these

4. For $x \in R$ and a continuous function f, let $I_1 = \int_{\sin^2 t}^{1+\cos^2 t} x\, f(x(2-x))dx$ and $I_2 = \int_{\sin^2 t}^{1+\cos^2 t} f(x(2-x))dx$. Then I_1 / I_2 is:

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

5. $\int_0^{n^2} [\sqrt{x}]dx$ is equal to (where [.] denotes greatest integer function) :

(A) $\frac{n(n+1)(4n+1)}{6}$ **(B)** $\frac{n(n-1)(4n+1)}{6}$ **(C)** $\frac{n(n-1)(4n-1)}{6}$ **(D)** None of these

6. The value of $\int_a^{a+\pi/2} (\sin^4 x + \cos^4 x)dx$ is :

(A) $3\pi/4$ **(B)** $a\left(\frac{\pi}{2}\right)^2$ **(C)** $3\pi/8$ **(D)** $\frac{3}{8}\pi a^2$

7. If $I = \int_{-2}^{0} [x^3 + 3x^2 + 3x + (x+1)\cos(x+1)]dx$, then I equals :

(A) –4 **(B)** –3 **(C)** –2 **(D)** –1

8. If $I = \int_2^3 \frac{2x^5 + x^4 - 2x^3 + 2x^2 + 1}{(x^2+1)(x^4-1)}dx$, then I equals:

(A) $\frac{1}{2}\log 6 + \frac{1}{10}$ **(B)** $\frac{1}{2}\log 6 - \frac{1}{10}$ **(C)** $\frac{1}{2}\log 3 - \frac{1}{10}$ **(D)** $\frac{1}{2}\log 2 + \frac{1}{10}$

9. If $I=\int_0^{\pi/2}\cos^n x\sin^n x\,dx\cdot=\lambda\int_0^{\pi/2}\sin^n x\,dx$ then λ equals :

(A) 2^{-n+1} **(B)** 2^{-n-1} **(C)** 2^{-n} **(D)** 2^{-1}

10. If $I=\int_0^{\pi/2}\frac{\sin 8x\log(\cot x)}{\cos 2x}dx$, then I equals:

(A) $-\pi/2$ **(B)** $\pi/3$ **(C)** $-\frac{1}{3}$ **(D)** 0

11. If $I=\int_{1/e}^{e}|\log x|\frac{dx}{x^2}$, then I equals:

(A) 2 **(B)** $2/e$ **(C)** $2(1-1/e)$ **(D)** 0

12. If $b>a$, and $I=\int_a^b\frac{dx}{\sqrt{(x-a)(b-x)}}$, then I equals :

(A) $\pi/2$ **(B)** π **(C)** $3\pi/2$ **(D)** 2π

13. If $b>a$, and $I=\int_a^b\sqrt{\frac{x-a}{b-x}}dx$, then I equals :

(A) $\frac{\pi}{2}(b-a)$ **(B)** $\pi(b-a)$ **(C)** $\pi/2$ **(D)** $2\pi(b-a)$

14. If $I=\int_0^{\pi/2}\frac{dx}{5+3\sin x}=\lambda\tan^{-1}\left(\frac{1}{2}\right)$, then value of λ is :

(A) 1 **(B)** $1/2$ **(C)** $1/3$ **(D)** $1/4$

15. If $0<\alpha<1$ and $I=\int_{-1}^{1}\frac{dx}{\sqrt{1-2\alpha x+\alpha^2}}$, then I equals :

(A) $\frac{1}{\alpha}$ **(B)** $\frac{2}{\alpha}$ **(C)** $\frac{3}{\alpha}$ **(D)** None of these

16. The value of the integral $I=\int_0^{\pi/4}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$ is :

(A) $\frac{1}{ab}\tan^{-1}\frac{b}{a}$ **(B)** $\frac{1}{ab}\tan^{-1}\frac{a}{b}$ **(C)** $ab\tan^{-1}\left(\frac{b}{a}\right)$ **(D)** $ab\tan^{-1}\left(\frac{a}{b}\right)$

17. If $\int_0^{x^2(1+x)}f(t)dt=x$ then $f(2)$ is equal to :

(A) $\frac{1}{3}$ **(B)** $\frac{1}{4}$ **(C)** 1 **(D)** $\frac{1}{5}$

18. The numbers A,B and C such that a function of the form $f(x)=Ax^2+Bx+C$ satisfies the conditions $f'(1)=8,\ f(2)+f''(2)=33$ and $\int_0^1 f(x)dx=7/3$, are :

(A) $A=1,B=-4,C=2$ **(B)** $A=7,B=-6,C=3$

(C) $A=8,B=-6,C=3$ **(D)** None of these

19. The area of the figure bounded by $y^2 = 2x + 1$ and $x - y - 1 = 0$ is :

(A) $\frac{2}{3}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{8}{3}$ **(D)** $\frac{16}{3}$

20. If $I_1 = \int_x^1 \frac{dt}{1+t^2}$ and $I_2 = \int_1^{1/x} \frac{dt}{1+t^2}$ for $x > 0$, then :

(A) $I_1 = I_2$ **(B)** $I_1 > I_2$ **(C)** $I_2 > I_1$ **(D)** $I_2 = \left(\frac{\pi}{2}\right) - \tan^{-1} x$

21. Area of the region $\left\{(x, y) : (x-1)^2 < y < |x-1|\right\}$ is :

(A) $1/3$ **(B)** $2/3$ **(C)** $4/3$ **(D)** $5/3$

22. Area enclosed by the curve $f(x) = \left(\frac{x^2}{4} - \frac{8}{x^2+4}\right)$, the x-axis and the ordinates $x = \pm 3$, equals :

(A) $\frac{4}{3} + 8\tan^{-1}\frac{3}{2} - 2\pi$ **(B)** $\frac{4}{3} - 8\tan^{-1}\frac{3}{2} - 2\pi$

(C) $\frac{4}{3} + 8\tan^{-1}\frac{3}{2} - \pi$ **(D)** None of these

***23.** If the function $f(x) = Ae^{2x} + Be^x + Cx$ satisfies the condition $f(0) = -1,\ f'(\log 2) = 31$ and $\int_0^{\log 4} \left(f(x) - cx\right)dx = \frac{39}{2}$, then :

(A) $A = 5$ **(B)** $B = -6$ **(C)** $C = 3$ **(D)** $B = 6$

***24.** If $I_1 = \int_0^1 2^{x^3} dx$, $I_2 = \int_0^1 2^{x^4} dx$, $I_3 = \int_1^2 2^{x^3} dx$, and $I_4 = \int_1^2 2^{x^4} dx$, then :

(A) $I_2 > I_1$ **(B)** $I_3 > I_4$ **(C)** $I_4 > I_3$ **(D)** $I_1 > I_2$

25. $\int_0^{\pi/2} \log\left(\tan^n x\right) dx$ is :

(A) $\frac{\pi}{2}$ **(B)** $\frac{\pi}{3}$ **(C)** 0 **(D)** $-\frac{\pi}{2}$

26. $\int_{-\pi/4}^{\pi/4} \frac{dx}{\sec^2 x\,(1 + \sin x)} =$

(A) $\pi/4$ **(B)** $\pi/2$ **(C)** π **(D)** 2π

27. Area included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is equal to :

(A) $\frac{8a^2}{3}$ **(B)** $\frac{16a^2}{3}$ **(C)** $\frac{4a^2}{3}$ **(D)** None of these

28. $\int_0^1 x(1-x)^{99}\,dx =$

(A) $\frac{11}{10100}$ **(B)** $\frac{1}{10010}$ **(C)** $\frac{1}{10100}$ **(D)** None of these

29. $\int_{1/4}^{4} \frac{1}{x}\tan\left(x-\frac{1}{x}\right)dx =$

(A) 1 **(B)** 0 **(C)** –1 **(D)** None of these

30. If $f(x) = \int_{x^2}^{x^2+4} e^{-t^2}dt$, then the function $f(x)$ increases in :

(A) $(-\infty, 0)$ **(B)** $(0, \infty)$ **(C)** $(-1, 2)$ **(D)** $(-2, \infty)$

31. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to :

(A) 5/3 **(B)** 1/3 **(C)** 2/3 **(D)** 4/3

32. Let $f(x) = \int_0^x \sqrt{6-x^2}\,dx$, then the real roots of the equation $x^2 - f'(x) = 0$ are :

(A) $x = \pm\sqrt{6}$ **(B)** $x = \pm\sqrt{3}$ **(C)** $x = \pm\sqrt{2}$ **(D)** $x = \pm 1$

33. Let f be an odd function, then $\int_{-1}^{1}\left(|x| + f(x)\cos x\right)dx$ is equal to :

(A) 0 **(B)** 1 **(C)** 2 **(D)** None of these

34. The value of the integral $\int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x}\,dx$ is :

(A) $log(2)$ **(B)** $log(3)$ **(C)** $\frac{1}{4}log(3)$ **(D)** $\frac{1}{8}log(3)$

35. The area of the plane figure bounded by the interval $[-5\pi/6, \pi]$ of the x-axis, the graph of the function $y = \cos x$ and the segment so the straight lines $x = -5\pi/6$ and $x = \pi$ is :

(A) 3/2 **(B)** 5/2 **(C)** 3/4 **(D)** 7/2

36. Suppose that the graph of $y = f(x)$ contains the points (0 , 4) and (2, 7). If f' is continuous then $\int_0^2 f'(x)dx$ is equal to:

(A) 2 **(B)** –2 **(C)** 3 **(D)** None of these

37. The value of $\lim_{x\to 0} \frac{\int_0^x \sin t^2 dt}{\sin x^2}$ is :

(A) 1 **(B)** 0 **(C)** 2 **(D)** None of these

38. The area between the curves $y = x^2$ and $y = x^{1/3}$ taking $x \in [-1, 1]$ is :

(A) 1/2 **(B)** 2 **(C)** 3/4 **(D)** 3/2

39. The value of the definite integral $\int_1^e \left((x+1)e^x \cdot \ell nx\right)dx$ is :

(A) e **(B)** e^{e+1} **(C)** $e^e(e-1)$ **(D)** $e^e(e-1)+e$

40. $\int_0^\infty \frac{x\tan^{-1}x}{\left(1+x^2\right)^2}dx$

(A) $\frac{\pi}{2}$ **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{6}$ **(D)** $\frac{\pi}{8}$

41. $\int_2^3 \frac{(x+2)^2}{2x^2-10x+53}dx$ is equal to:

(A) 2 **(B)** 1 **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{2}$

42. $\int_2^4 \left(\log_x 2 - \frac{(\log_x 2)^2}{\ell n 2}\right)dx$ equals to:

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4

43. The value of $\int_1^2 \left(\left[x^2\right]-[x]^2\right)dx$, where [.] denotes the greatest integer function, is equal to :

(A) $4+\sqrt{2}-\sqrt{3}$ **(B)** $4-\sqrt{2}+\sqrt{3}$ **(C)** $4-\sqrt{3}-\sqrt{2}$ **(D)** None of these

44. The area of the closed figure bounded by $x=-1, x=2$ and $y=\begin{cases}-x^2+2 & , \quad x \le 1 \\ 2x-1 & , \quad x>1\end{cases}$ and the abscissa axis is:

(A) $16/3$ sq. units **(B)** $10/3$ sq. units **(C)** $13/3$ sq. units **(D)** $7/3$ sq. units

45. The area of the region for which $0<y<3-2x-x^2$ and $x>0$ is :

(A) $\int_1^3 \left(3-2x-x^2\right)dx$ **(B)** $\int_0^3 \left(3-2x-x^2\right)dx$

(C) $\int_0^1 \left(3-2x-x^2\right)dx$ **(D)** $\int_{-1}^3 \left(3-2x-x^2\right)dx$

46. The area bounded by the curves $\sqrt{x}+\sqrt{y}=1$ and $x+y=1$ is :

(A) $\frac{1}{3}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{2}$ **(D)** None of these

47. The area $\left\{(x, y)\,;\, x^2 \le y \le \sqrt{x}\right\}$ is equal to :

(A) $1/3$ **(B)** $2/3$ **(C)** $1/6$ **(D)** None of these

48. The value of $\int_0^{\pi/4} \frac{dx}{\cos^4 x-\cos^2 x\sin^2 x+\sin^4 x}dx$

(A) 0 **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{2}$ **(D)** None of these

49. The value of $\int_1^{e^{37}} \frac{\pi \sin(\pi \ln x)}{x} dx$ is :

(A) 0 **(B)** 1 **(C)** 2 **(D)** None of these

50. If $\int_0^{\pi/2} \cos^m x \sin^m x \, dx = \lambda \int_0^{\pi/2} \sin^m x \, dx$ then value of λ is :

(A) 2^m **(B)** 2^{-m} **(C)** $2^{m/2}$ **(D)** $2^{-m/2}$

51. $\int_0^a x^{5/2} \sqrt{a-x} \, dx =$

(A) $\frac{5\pi a}{128}$ **(B)** $\frac{7\pi a}{128}$ **(C)** $\frac{7\pi a^4}{128}$ **(D)** $\frac{5\pi a^4}{128}$

52. If $y = \int_{\pi^2/16}^{x^2} \frac{\cos x \cos\sqrt{\theta}}{1+\sin^2\sqrt{\theta}} d\theta$ then $\frac{dy}{dx}$ at $x = \pi$ is :

(A) π **(B)** 2π **(C)** 3π **(D)** None of these

53. $\int_{-3\pi/2}^{-\pi/2} \left[(x+\pi)^3 + \cos^2(x+3\pi)\right] dx =$

(A) $\frac{\pi^4}{32} + \frac{\pi}{2}$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{\pi}{4} - 1$ **(D)** $\frac{\pi^4}{32}$

54. The function $F(x) = \int_0^x \log\frac{(1-x)}{(1+x)} dx$ is a function which is :

(A) even **(B)** odd **(C)** periodic **(D)** None of these

55. Let $f : R \to R$ be a differentiable function and $f(1) = 1,\ f'(1) = 3$. Then the value of $\lim_{x \to 1} \frac{\int_1^{x^2} (f(t) - t) dt}{(x-1)^2}$ is:

(A) 3 **(B)** 4 **(C)** 0 **(D)** 2

56. The line $y = mx$ bisects the area enclosed by the curve $y = 1 + 4x - x^2$ and the lines $x = 0, x = \frac{3}{2}$ and $y = 0$. Then the value of m is :

(A) $\frac{13}{6}$ **(B)** $\frac{6}{13}$ **(C)** $\frac{3}{2}$ **(D)** 4

57. Let $I_1 = \int_1^2 \frac{1}{\sqrt{(1+x^2)}} dx$ and $I_2 = \int_1^2 \frac{1}{x} dx$, then :

(A) $I_1 > I_2$ **(B)** $I_1 < I_2$ **(C)** $I_1 = I_2$ **(D)** None of these

58. The value of $\lim_{n\to\infty}\sum_{r=1}^{4n}\frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+4\sqrt{n}\right)^2}$ is equal to:

(A) $\frac{1}{35}$ **(B)** $\frac{1}{14}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{1}{5}$

59. If $\int_0^\infty \frac{x^2dx}{\left(x^2+a^2\right)\left(x^2+b^2\right)\left(x^2+c^2\right)}=\frac{\pi}{2\left(a+b\right)\left(b+c\right)\left(c+a\right)}$, then the value of $\int_0^\infty \frac{dx}{\left(x^2+4\right)\left(x^2+9\right)}$ is:

(A) $\frac{\pi}{60}$ **(B)** $\frac{\pi}{20}$ **(C)** $\frac{\pi}{40}$ **(D)** $\frac{\pi}{80}$

60. If $f(x)$ satisfies the condition of Rolle's theorem in [1, 2], then $\int_1^2 f'(x)dx$ is equal to:

(A) 1 **(B)** 3 **(C)** 0 **(D)** None of these

61. If $f(x)$ and $g(x)$ are continuous functions, then $\int_{\ln\lambda}^{\ln\frac{1}{\lambda}}\frac{f\left(x^2/4\right)\left[f\left(x\right)-f\left(-x\right)\right]}{g\left(x^2/4\right)\left[g\left(x\right)+g\left(-x\right)\right]}dx$ is:

(A) Dependent on λ **(B)** A non-zero constant

(C) Zero **(D)** None of these

62. $\int_0^4 \frac{\left(y^2-4y+5\right)\sin\left(y-2\right)dy}{\left(2y^2-8y+11\right)}$ is equal to:

(A) 0 **(B)** 2 **(C)** -2 **(D)** None of these

63. Let $I_1=\int_0^1\frac{e^xdx}{1+x}$, $I_2=\int_0^1\frac{x^2dx}{e^{x^3}(2-x^3)}$. Then $\frac{I_1}{I_2}$ is equal to:

(A) $3/e$ **(B)** $e/3$ **(C)** $3e$ **(D)** $1/3e$

64. If $f\left(x\right)=\int_0^1\frac{dt}{1+\left|x-t\right|}$, then $f'\left(\frac{1}{2}\right)$ is equal to:

(A) 0 **(B)** $1/2$ **(C)** 1 **(D)** None of these

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65. If $f(x)=\int_0^x \sin^4 t\, dt$, then $f(x+\pi)$ is equal to :

(A) $f(\pi)$ **(B)** $f(x)$ **(C)** $f(x)+f(\pi)$ **(D)** $f(x)\cdot f(\pi)$

66. The interval [a, b] such that the value of $\int_a^b (2+x-x^2)dx$ is maximum :

(A) [–1, 2] **(B)** [–2, 1] **(C)** [–2, –1] **(D)** [1, 2]

67. If $f(x)$ is a function satisfying $f\left(\frac{1}{x}\right)+x^2 f(x)=0$ for all non-zero x, then $\int_{\sin\theta}^{\text{cosec}\theta} f(x)dx$ equals :

(A) $\sin\theta+\text{cosec}\theta$ **(B)** $\sin^2\theta$ **(C)** $\text{cosec}^2\theta$ **(D)** None of these

68. If $I=\int_0^{2\pi} e^{x/2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$, then I equals :

(A) $\frac{\pi}{2}$ **(B)** 0 **(C)** $-\pi/2$ **(D)** π

69. If $I_1=\int_0^{\pi/2} f(\sin 2x)\sin x\, dx$ and $I_2=\int_0^{\pi/4} f(\cos 2x)\cos x\, dx$, then $\frac{I_1}{I_2}$ equals :

(A) 1 **(B)** $\frac{1}{\sqrt{2}}$ **(C)** $\sqrt{2}$ **(D)** 2

70. $\int_0^1 \frac{dx}{(1+x)(2+x)\sqrt{x(1-x)}}$ equals :

(A) 2π **(B)** π **(C)** $\frac{\pi}{2}$ **(D)** $\frac{\pi}{\sqrt{6}}(\sqrt{3}-1)$

71. If $I=\int_{-1}^{2} |x\sin\pi x|\, dx$, then I equals:

(A) $\frac{1}{\pi}$ **(B)** $\frac{2}{\pi}$ **(C)** $\frac{4}{\pi}$ **(D)** $\frac{5}{\pi}$

72. A polynomial P is positive for $x>0$ and the area of the region bounded by $P(x)$, the x-axis, and the vertical lines $x=0$ and $x=k$ is $\frac{k^2(k+3)}{3}$, then the polynomial $P(x)$ is :

(A) x^2+x+1 **(B)** x^2+2x+1 **(C)** x^2+2x **(D)** x^2+1

73. If $m\neq n, m, n\in N$ then the value of $\int_0^{2\pi}\cos mx\cos nx.dx$ is :

(A) 0 **(B)** 2π **(C)** π **(D)** dependent on m and n

74. Let f be a continuous function on R satisfying $f(x+y)=f(x)+f(y)$ for all $x, y \in R$ with $f(1)=2$ and g be a function satisfying $f(x)+g(x)=e^x$ then the value of the integral $\int_0^1 f(x)g(x)dx$ is :

(A) $\frac{1}{e}-4$ **(B)** $\frac{1}{4}(e-2)$ **(C)** $2/3$ **(D)** $\left(\frac{1}{2}\right)(e-3)$

75. A line tangent to the graph of the function $y=f(x)$ at the point $x=a$ forms an angle $\frac{\pi}{3}$ with y-axis and at $x=b$ an angle of $\frac{\pi}{4}$ with x-axis, then $\left|\int_a^b f''(x)dx\right|$ is :

(A) $\frac{1}{\sqrt{3}}-1$ **(B)** $-\frac{\pi}{12}$ **(C)** $\frac{\pi}{12}$ **(D)** $\sqrt{3}-1$

76. If $f(x)=\int_1^x \frac{\log t}{t+1}dt$ and $f(x)+f\left(\frac{1}{x}\right)=k(\log x)^2$, then k is equal to :

(A) 1 **(B)** $1/2$ **(C)** $1/4$ **(D)** $1/3$

77. Let $f: R \to R$ and $g: R \to R$ be continuous functions. Then the value of the integral

$$\int_{-\pi/2}^{\pi/2} \left[f(x)+f(-x)\right]\left[g(x)-g(-x)\right] dx =$$

(A) 1 **(B)** 0 **(C)** $f\left(\frac{\pi}{2}\right)+f\left(-\frac{\pi}{2}\right)$ **(D)** $g\left(\frac{\pi}{2}\right)+g\left(-\frac{\pi}{2}\right)$

78. Let $g(x)$ be a function satisfying $g'(x)=g(x)$ and $g(0)=1$ and $f(x)$ be a function that satisfies $f(x)+g(x)=x^2$. Then the value of the integral $\int_0^1 f(x)g(x)dx$ is :

(A) $\frac{e-7}{4}$ **(B)** $\frac{e-3}{2}$ **(C)** $e-\frac{e^2}{2}-\frac{3}{2}$ **(D)** $e+\frac{e^2}{2}+\frac{3}{2}$

79. If $I_1=\int_e^{e^2} \frac{dx}{\log x}$ and $I_2=\int_1^2 \frac{e^x}{x}dx$, then :

(A) $I_1=2I_2$ **(B)** $I_2=2I_1$ **(C)** $I_1+I_2=0$ **(D)** $I_1-I_2=0$

80. $\int_{-1}^{1} \frac{\left(\sin^{-1}x\right)^2 dx}{1+\pi^{\sin x}}$ is :

(A) $\frac{\pi^2-8}{4}$ **(B)** $\frac{\pi^2+8}{4}$ **(C)** $\frac{\pi^2-8}{2}$ **(D)** $\frac{\pi^2+8}{2}$

81. Let $f(x)$ be differentiable in R and $f(x)=2+x^2\int_0^2 f(t)dt+\int_0^2 tf(t)dt$. Then $\int_0^1 f(x)dx$ is :

(A) $\frac{6}{19}$ **(B)** $\frac{3}{19}$ **(C)** $\frac{14}{19}$ **(D)** $\frac{-6}{19}$

82. If $g(x)=\frac{1}{x}\int_2^x \left[3t-2g'(t)\right]dt$, then $g'(2)$ is :

(A) $\frac{1}{2}$ **(B)** $\frac{3}{4}$ **(C)** 1 **(D)** $\frac{3}{2}$

83. If $f(x)=\min,\{2\sin x, 1-\cos x, 1\}$, then $\int_0^\pi f(x)dx=$

(A) $\sqrt{3}-1+\frac{5\pi}{6}$ **(B)** $\sqrt{3}-1+\frac{2\pi}{3}$ **(C)** $1-\sqrt{3}+\frac{2\pi}{3}$ **(D)** $1-\sqrt{3}+\frac{5\pi}{6}$

84. $\int_{a+c}^{b+c} f(x-c)dx=$

(A) $\int_a^b f(x-c)dx$ **(B)** $\int_a^b f(x)dx$

(C) $\int_a^b f(a+b+c+x)dx$ **(D)** None of these

85. The value of $\int_0^\pi \frac{x\ dx}{4\cos^2 x+9\sin^2 x}$ is equal to :

(A) $\pi^2/12$ **(B)** $\pi^2/4$ **(C)** $\pi^2/6$ **(D)** $\pi^2/3$

86. An inflection point on the graph of the function $y=\int_0^x (t-1)(t-2)^2\ dt$ is :

(A) $x=-1$ **(B)** $x=3/2$ **(C)** $x=4/3$ **(D)** $x=1$

87. Let f be a periodic continuous function with period $T>0$. If $I=\int_0^T f(x)dx$, then the value of $I_1=\int_4^{4+4T} f(3x)dx$ is :

(A) I **(B)** $2I$ **(C)** $3I$ **(D)** $4I$

88. The value of $\int_0^2 x^{\left[x^2+1\right]}dx$, where [x] is the greatest integer less than or equal to x is :

(A) 2 **(B)** 8/3 **(C)** 4 **(D)** None of these

89. The area bounded by the curves $y=\sqrt{5-x^2}$ and $y=|x-1|$ is :

(A) $\left(\frac{5\pi}{4}-2\right)$ **(B)** $\left(\frac{5\pi-2}{4}\right)$ **(C)** $\left(\frac{5\pi-2}{2}\right)$ **(D)** $\left(\frac{\pi}{2}-5\right)$

90. If $f(x)=\begin{vmatrix} \sin x+\sin 2x+\sin 3x & \sin 2x & \sin 3x \\ 3+4\sin x & 3 & 4\sin x \\ 1+\sin x & \sin x & 1 \end{vmatrix}$ then the value of $\int_0^{\pi/2} f(x)dx$ is :

(A) 3 **(B)** 0 **(C)** 2/3 **(D)** 1/3

91. The area bounded by the curve $y = f(x) = x^4 - 2x^3 + x^2 + 3$, x-axis and the ordinates corresponding to minimum of the function $f(x)$ is :

(A) 1 **(B)** $\frac{91}{30}$ **(C)** $\frac{30}{9}$ **(D)** 4

92. If $f(x) = \begin{cases} 3[x] - 5\frac{|x|}{x} & , \; x \neq 0 \\ 2 & , \; x = 0 \end{cases}$ then $\int_{-3/2}^{2} f(x)dx$ is equal to : ([.] denotes the greatest integer function)

(A) $-\frac{11}{2}$ **(B)** $-\frac{7}{2}$ **(C)** -6 **(D)** $-\frac{17}{2}$

93. The value of $\int_{-10}^{20} \left[\cot^{-1} x\right] dx$ is : (where [.] is the greatest integer function] = $A + \cot B + \cot C + \cot D$ then $A + B + C + D$ is :

(A) 36 **(B)** 35 **(C)** 38 **(D)** 40

94. $\int_{0}^{\pi/2} \left(2\log\sin x - \log\sin 2x\right)dx$ equals :

(A) $\pi\log 2$ **(B)** $-\pi\log 2$ **(C)** $\left(\frac{\pi}{2}\right)\log 2$ **(D)** $-\left(\frac{\pi}{2}\right)\log 2$

95. $\int_{1/2}^{2} \frac{1}{x}\sin\left(x - \frac{1}{x}\right)dx$ has the value equal to:

(A) 0 **(B)** $3/4$ **(C)** $5/4$ **(D)** 2

96. Evaluate : $\int_{0}^{\pi/2} \frac{a\sin x + b\cos x}{\sin\left(\frac{\pi}{4} + x\right)}dx$

(A) $\frac{\pi(a+b)}{2}$ **(B)** $\frac{\pi(a+b)}{2\sqrt{2}}$ **(C)** $\frac{\pi(a-b)}{2\sqrt{2}}$ **(D)** None of these

97. Evaluate : $\int_{0}^{1} \frac{\sin^{-1}\sqrt{x}}{x^2 - x + 1}dx$

(A) $\frac{\pi}{6\sqrt{3}}$ **(B)** $\frac{\pi^2}{\sqrt{3}}$ **(C)** $\frac{\pi^2}{6}$ **(D)** $\frac{\pi^2}{6\sqrt{3}}$

98. The value of the integral $\int_{\pi/3}^{\pi/2} x\sin\left(\pi[x] - x\right)dx$ is : (where [x] denotes greatest integer function)

(A) $\frac{1}{2} + \frac{\pi}{6}$ **(B)** $-\frac{1}{2} - \frac{\pi}{6}$ **(C)** $1 - \frac{\sqrt{3}}{2} + \frac{\pi}{6}$ **(D)** $\frac{\sqrt{3}}{2} - 1 - \frac{\pi}{6}$

99. $f(x) = \min\left\{x + 1, 2\,\text{sgn}\left(|x|\right)\right\}, \forall\, x \in R$, then $\int_{-5}^{4} f(x)dx =$

(A) 1 **(B)** 0 **(C)** 3 **(D)** 5

100. The value of $\int_{-1}^{3}\left(\tan^{-1}\frac{x}{x^2+1}+\tan^{-1}\frac{x^2+1}{x}\right)dx$

(A) 0 **(B)** π **(C)** $-\pi$ **(D)** None of these

101. The value of $\int_{-1}^{1}\max\left(|x|, x^2, x^4\right)dx$ is equal to :

(A) 1 **(B)** 2 **(C)** 3 **(D)** None of these

102. For $\theta \in (0,\pi)\cup(\pi, 2\pi)$ $I_1=\int_0^{\infty}\frac{dx}{x^2+2x\cos\theta+1}$ and $I_2=2\int_0^{1}\frac{dx}{x^2+2x\cos\theta+1}$ then $\frac{I_1}{I_2}=.$

(A) 1 **(B)** 2 **(C)** $1/2$ **(D)** None of these

103. If $\int_{\log 2}^{x}\frac{dy}{\sqrt{(e^y-1)}}=\frac{\pi}{6}$ then the value of 'x' is :

(A) $x=\log 2$ **(B)** $x=2\log 2$ **(C)** $x=3\log 2$ **(D)** None of these

104. The value of $\int_0^1\log\left[\sqrt{1-x}+\sqrt{1+x}\right]dx$ is :

(A) $\frac{1}{2}\log 2+\frac{\pi}{2}$ **(B)** $\log 2-\frac{\pi}{2}-\frac{1}{2}$ **(C)** $\frac{1}{4}\log 2+\frac{\pi}{2}-\frac{1}{4}$ **(D)** $\frac{1}{2}\log 2+\frac{\pi}{2}-\frac{1}{2}$

105. If $I_m=\int_1^e(\log x)^m\,dx$, then the value $I_m+m\,I_{m-1}$ is :

(A) 0 **(B)** $-e$ **(C)** e **(D)** None of these

106. If $I=\int_0^1\frac{e^t}{1+t}\,dt$, then $P=\int_0^1 e^t\log(1+t)\,dt=$

(A) I **(B)** $2I$ **(C)** $e\log 2-I$ **(D)** None of these

107. $I=\int_0^{\pi/4}\frac{\sin x+\cos x}{\cos^2 x+\sin^4 x}\,dx=\frac{1}{a}\log\frac{(a+1)}{\sqrt{2}}+\frac{\pi}{4}$ where 'a' is :

(A) $\sqrt{2}$ **(B)** $\sqrt{3}$ **(C)** $\sqrt{5}$ **(D)** None of these

108. The value of $\int_0^1\frac{2-x^2}{(1+x)\sqrt{(1-x^2)}}dx$ is:

(A) $\frac{\pi}{2}$ **(B)** $-\frac{\pi}{2}$ **(C)** 0 **(D)** None of these

109. The value of $\int_{\sqrt{\left(\frac{3a^2+b^2}{4}\right)}}^{\sqrt{\left(\frac{a^2+b^2}{2}\right)}}\frac{x}{\left\{\sqrt{(x^2-a^2)(b^2-x^2)}\right\}}dx$ is :

(A) $\frac{\pi}{6}$ **(B)** $\frac{\pi}{12}$ **(C)** $\frac{\pi}{4}$ **(D)** $\frac{\pi}{2}$

110. The value of $\int_0^{\pi} \sin^3 x \left(1+2\cos x\right)\left(1+\cos x\right)^2 dx$ is :

(A) $\frac{2}{3}$ **(B)** $\frac{4}{3}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{8}{3}$

111. Evaluate : $\int_0^{\pi/2} \frac{\cos^9 x}{\cos^3 x + \sin^3 x} dx$

(A) $\frac{1}{2}\left[\frac{5\pi}{16} - \frac{1}{12}\right]$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{5\pi}{16}$ **(D)** None of these

112. The value of $\int_0^{\pi/6} \cos^4 3\theta \sin^2 6\theta$ is :

(A) $\frac{5\pi}{12}$ **(B)** $\frac{7\pi}{192}$ **(C)** $\frac{5\pi}{192}$ **(D)** None of these

113. The value of $\int_0^{\pi/2} \sin x \log\left(\sin x\right) dx$ is :

(A) $\log_e e$ **(B)** $\log_e 2$ **(C)** $\log_e\left(\frac{e}{2}\right)$ **(D)** $\log_e\left(\frac{2}{e}\right)$

114. The value of $\int_0^{\infty} \log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2}$ is :

(A) $\log 2$ **(B)** $\frac{1}{\pi}\log 2$ **(C)** $\pi \log 2$ **(D)** None of these

***115.** The value of $\int_0^{\pi} \log\left(1+\cos x\right) dx$ is :

(A) $\log\frac{1}{2}$ **(B)** $\pi \log 2$ **(C)** $-\pi \log 2$ **(D)** $\pi \log \frac{1}{2}$

116. $\int_{1/e}^{\tan t} \frac{x\,dx}{1+x^2} + \int_{1/e}^{\cot t} \frac{dx}{x\left(1+x^2\right)} dx$ is:

(A) –1 **(B)** 1 **(C)** 0 **(D)** 2

117. The area bounded by the x-axis ; part of the curve $y = \left(1+\frac{8}{x^2}\right)$ and the ordinates at x = 2 and x = 4. If the ordinate at $x = a$ divides the area into two parts, then the value of 'a' is :

(A) $\sqrt{2}$ **(B)** $3\sqrt{2}$ **(C)** $4\sqrt{2}$ **(D)** $2\sqrt{2}$

118. The area included between the parabola $y = \frac{x^2}{4a}$ and the curve $y = \frac{8a^3}{x^2+4a^2}$.

(A) $a^2\left[2\pi - \frac{4}{3}\right]$ **(B)** $2a^2\pi$ **(C)** $a^2\frac{\pi}{3}$ **(D)** None of these

119. The area bounded by the curve $y = \left(x-1\right)\left(x-2\right)\left(x-3\right)$ lying between the ordinates x = 0 and x = 3 is:

(A) $\frac{3}{4}$ **(B)** $\frac{11}{4}$ **(C)** $\frac{5}{4}$ **(D)** None of these

120. Let $f(x)=\begin{cases}2x-1 & , -3\le x<1\\ 3x^2-2 & , x\ge 1\end{cases}$ and $g(x)=\begin{cases}4x+7 & , -5\le x<0\\ 5x^2-x+7 & , x\ge 0\end{cases}$.

Then value of $\int_{-2}^{2}(gof)(x)dx$ equals :

(A) 0 **(B)** 101/12 **(C)** 1991/6 **(D)** 1991/12

121. Evaluate : $\int_0^{\pi}\frac{x^3\cos^4 x\sin^2 x}{\left(\pi^2-3\pi x+3x^2\right)}dx$

(A) $\frac{\pi}{32}$ **(B)** $\frac{\pi}{36}$ **(C)** $\frac{\pi^2}{32}$ **(D)** $\frac{\pi^2}{36}$

122. $\int_{e^{e^e}}^{e^{e^{e^e}}}\frac{dx}{x\ell nx\,.\,\ell n(\ell nx)\,.\,\ell n(\ell n(\ell nx))}$ equals :

(A) 1 **(B)** $\frac{1}{e}$ **(C)** $e-1$ **(D)** $1+e$

123. Evaluate : $\int_0^1\frac{1-x}{1+x}\cdot\frac{dx}{\sqrt{x+x^2+x^3}}$

(A) $\frac{\pi}{2}$ **(B)** $\frac{\pi}{4}$ **(C)** $\frac{\pi}{3}$ **(D)** None of these

124. If $\int_0^a\frac{dx}{\sqrt{x+a}+\sqrt{x}}=\int_0^{\pi/8}\frac{2\tan\theta}{\sin 2\theta}d\theta$, then value of a is equal to : (a > 0)

(A) $\frac{3}{4}$ **(B)** $\frac{\pi}{4}$ **(C)** $\frac{3\pi}{4}$ **(D)** $\frac{9}{16}$

125. If $f(x)$ is a continuous function for all real values of x and satisfies $\int_n^{n+1}f(|x|)dx=\frac{n^2}{2}, \forall\, n\in I$ then $\int_{-3}^{5}f(|x|)dx$ is :

(A) 10 **(B)** 20 **(C)** 22 **(D)** 18

126. If $\int_n^{n+1}\left[x+\left[x+[x]\right]\right]dx=kn, n\in I$ (where $[\cdot]$ denotes the greatest integer function), then k is/are :

(A) $k=1$ **(B)** $k=2$ **(C)** $k=4$ **(D)** $k=3$

127. Evaluate : $\int_0^{\infty}\frac{x\log x}{\left(1+x^2\right)^2}dx$

(A) 1 **(B)** 0 **(C)** –1 **(D)** None of these

128. Given $\int_0^{\pi/2} \frac{dx}{1+\sin x+\cos x} = \ell n 2$, then the value of $\int_0^{\pi/2} \frac{\sin x}{1+\sin x+\cos x} dx$ is equal to :

(A) $\frac{1}{2}\ell n 2$ **(B)** $\frac{\pi}{2}-\ell n 2$ **(C)** $\frac{\pi}{4}-\frac{1}{2}\ell n 2$ **(D)** $\frac{\pi}{2}+\ell n 2$

129. The value of $\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{\cos^{-1}\left(\frac{2x}{x^2+1}\right)+\tan^{-1}\left(\frac{2x}{1-x^2}\right)}{e^x+1} dx$ is :

(A) $\frac{\pi}{\sqrt{3}}$ **(B)** $\frac{\pi}{2\sqrt{3}}$ **(C)** $\frac{\pi}{2}$ **(D)** None of these

130. If $\int_0^1 \tan^{-1} x\, dx = \alpha$, then $\int_0^{\pi/4} \tan^{-1}\left(\frac{2\cos^2\theta}{2-\sin 2\theta}\right)\sec^2\theta\, d\theta$ is equal to :

(A) α **(B)** $\frac{\alpha}{2}$ **(C)** 3α **(D)** 2α

131. The value of $\int_{-(\pi/4)^{1/3}}^{(\pi/4)^{1/3}} \frac{x^2}{\left(1+\sin^2 x^3\right)\left(1+e^{x^7}\right)} dx$

(A) $\frac{1}{3\sqrt{2}}\tan^{-1}\sqrt{2}$ **(B)** $\frac{1}{3\sqrt{2}}\tan^{-1}\sqrt{2}$ **(C)** $\frac{1}{\sqrt{2}}\tan^{-1}\sqrt{2}$ **(D)** None of these

132. The value of $\int_a^{\infty} \frac{dx}{x^4\sqrt{\left(a^2+x^2\right)}} =$

(A) $\frac{2+\sqrt{2}}{3a^4}$ **(B)** $\frac{2-\sqrt{2}}{3a^4}$ **(C)** $\frac{2-\sqrt{2}}{a}$ **(D)** None of these

133. If $I_1 = \int_0^{\pi/2} \frac{x}{\sin x} dx$ and $I_2 = \int_0^1 \frac{\tan^{-1} x}{x} dx$, then $\frac{I_1}{I_2} =$

(A) $\frac{1}{2}$ **(B)** 1 **(C)** 2 **(D)** $\frac{\pi}{2}$

134. If $I_{10} = \int_0^{\pi/2} x^{10} \sin x\, dx$ then the value of $I_{10} + 90\, I_8$ is :

(A) $\frac{\pi}{2}$ **(B)** π **(C)** π^{10} **(D)** $10\left(\frac{\pi}{2}\right)^9$

135. If $I_n = \int_0^1 x^n \tan^{-1} x\, dx$, then $(n+1)I_n + (n-1)I_{n-2}$ is :

(A) $\pi - n$ **(B)** $\frac{\pi}{2}-\frac{1}{n}$ **(C)** $\frac{\pi}{2}-n$ **(D)** None of these

136. The whole area contained between the curve $y^2(a-x)=x^2(a+x)$ and its asymptotes.

(A) $2a\left(1+\frac{\pi}{4}\right)$ **(B)** $2a\,\pi$ **(C)** $2a(\pi+4)$ **(D)** None of these

137. The whole area contained between the curve $x^2(x^2+y^2)=a^2(y^2-x^2)$ and its asymptotes is :

(A) $2a^2\pi$ **(B)** $2a^2\left(\frac{\pi}{2}+\frac{1}{2}\right)$ **(C)** $2a^2\left(\frac{\pi}{2}+1\right)$ **(D)** None of these

138. If the value of $I_m=\int_0^{2a} x^m\sqrt{2ax-x^2}\,dx$ is :

(A) $I_m=\frac{(4m+1)a}{m+1}I_{m-1}$ **(B)** $I_m=\frac{(2m+1)a}{m+2}I_{m-1}$

(C) $I_m=\frac{(2m+1)a}{m+4}I_{m-1}$ **(D)** None of these

***139.** [.] represents greatest integer function : $\int_0^x [t]\,dt=\int_0^{[x]} t\,dt$, if :

(A) $x=5/2$ **(B)** $x=6$ **(C)** $x=-7/2$ **(D)** $x=101/2$

140. If $f(x)=x+\int_0^1 (xy^2+x^2y)f(y)dy$ and $f(x)=\frac{Ax^2+Bx}{C}$ then :

(A) A = 80 **(B)** B = 180 **(C)** C = 199 **(D)** C = 119

Paragraph for Questions 141 – 142

Let $f(x)$ be a continuous function defined on the closed interval [a, b], then $\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{1}{n}f\left(\frac{r}{n}\right)=\int_0^1 f(x)dx$

On the basis of above information, answer the following questions:

141. The value of $\lim_{n\to\infty}\left\{\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+......+\frac{1}{8n}\right\}$ is :

(A) 5/4 **(B)** 3/4 **(C)** 5/8 **(D)** 3/8

142. The value of $\lim_{n\to\infty}\left\{\prod_{r=1}^{n}\left(1+\frac{r}{n}\right)\right\}^{1/n}$ is :

(A) $3/e$ **(B)** $4/e$ **(C)** $1/e$ **(D)** $2/e$

Paragraph for Questions 143 – 144

Consider the function $h(n)=\int_0^a f(x)g(nx)dx$

143. Let $f(x)=2,\ g(x)=|\sin\pi x|$, and $n=\frac{1}{N}$, where N is natural number. If $\frac{h(n)}{a}$ is independent of n for $N=1, 2, 3, 4$ then minimum value of a is :

(A) 2 **(B)** 3 **(C)** 4 **(D)** 12

144. If $f(x)=\sin^2 x,\ g(x)=|\cos x|$ then $\lim\limits_{a\to\infty}\dfrac{h(1)}{a}$ is equal to :

(A) $2/3\pi$ **(B)** $1/3\pi$ **(C)** $1/2\pi$ **(D)** $1/\pi$

145. If the value of definite integral $\dfrac{18}{\pi^2}\displaystyle\int_{-\pi/4}^{\pi/4}\dfrac{x+\pi/4}{2-\cos 2x}dx$ is equal to $\sqrt{n}$, then the value of n is :

(A) 6 **(B)** $\sqrt{3}$ **(C)** 3 **(D)** $\sqrt{6}$

146. The value of $\lim\limits_{x\to 0}\left(\dfrac{1}{x^5}\displaystyle\int_0^x e^{-t^2}dt-\dfrac{1}{x^4}+\dfrac{1}{3x^2}\right)$ is :

(A) $-1/5$ **(B)** $1/10$ **(C)** $1/5$ **(D)** $1/2$

<u>Paragraph for Questions 147 – 148</u>

Suppose a and b are positive real numbers such that $ab=1$. Let for any real parameter t, the distance from the origin to the line $(ae^t)x+(be^{-t})y=1$ be denoted by $D(t)$ such that $I=\displaystyle\int_0^1\dfrac{dt}{[D(t)]^2}$, then :

147. The value of the definite integral I is :

(A) $\dfrac{e^2-1}{2}\left(b^2+\dfrac{a^2}{e^2}\right)$ **(B)** $\dfrac{e^2+1}{2}\left(a^2+\dfrac{b^2}{e^2}\right)$

(C) $\dfrac{e^2-1}{2}\left(a^2+\dfrac{b^2}{e^2}\right)$ **(D)** $\dfrac{e^2+1}{2}\left(b^2+\dfrac{a^2}{e^2}\right)$

148. The value of 'b' at which I is minimum, is :

(A) e **(B)** $1/e$ **(C)** $1/\sqrt{e}$ **(D)** $\sqrt{e}$

149. If $f:R\to R;\ f(x)=\sin x+x,$ then the value of $\int_0^{\pi}\left[f^{-1}(x)\right]dx,$ is equal to :

(A) $\dfrac{\pi^2}{2}-2$ **(B)** π^2 **(C)** $\dfrac{\pi^2}{2}+2$ **(D)** None of these

150. The value of $\displaystyle\int_0^{e-1}\dfrac{e^{\frac{x^2+2x-1}{2}}}{(x+1)}dx+\int_1^e x\log x . e^{\frac{x^2-2}{2}}dx$, is equal to :

(A) $(\sqrt{e})^{(e^2+1)}$ **(B)** $(\sqrt{e})^{e^2-1}$ **(C)** 0 **(D)** $(\sqrt{e})^{e^2-2}$

151. Let $a_n=\int_0^{\pi/2}(1-\sin t)^n\sin 2t,$ then $\lim\limits_{n\to\infty}\displaystyle\sum_{n=1}^{n}\dfrac{a_n}{n}$ is equal to

(A) $1/2$ **(B)** 1 **(C)** $4/3$ **(D)** $3/2$

152. The value of $\int_0^2 f(x)dx,$ where $f(x)=\begin{cases}0, & \text{when } x=\dfrac{n}{n+1}, n=1,2,3,\ldots\\ 1, & \text{else where}\end{cases}$ is equal to :

(A) 1 **(B)** 2 **(C)** 3 **(D)** None of these

153. The values of $\int_{-a}^{a}(\cos^{-1}x-\sin^{-1}\sqrt{1-x^2})dx$ is $(a>0)$ (where $\int_0^a \cos^{-1}x\,dx = A$) is :

(A) $\pi a - A$ **(B)** $\pi a + 2A$ **(C)** $\pi a - 2A$ **(D)** $\pi a + A$

154. If $f(x)=\begin{vmatrix} \cos x & e^{x^2} & 2x\cos^2 x/2 \\ x^2 & \sec x & \sin x + x^3 \\ 1 & 2 & x+\tan x \end{vmatrix}$, then the value of $\int_{-\pi/2}^{\pi/2}(x^2+1)(f(x)+f''(x))dx$, is equal to :

(A) 1 **(B)** –1 **(C)** 2 **(D)** None of these

155. The function $f(x)=\int_0^{x\in(0,2\pi)}\log_{|\sin t|}\left(\sin t+\frac{1}{2}\right)dt$ strictly increases in the interval :

(A) $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$ **(B)** $\left(\frac{5\pi}{6},2\pi\right)$ **(C)** $\left(\frac{\pi}{6},\frac{7\pi}{6}\right)$ **(D)** $\left(\frac{5\pi}{6},\frac{7\pi}{6}\right)$

156. Let $f:(0,\infty)\to R$ and $F(x)=\int_0^x t\,f(t)dt$. If $F(x^2)=x^4+x^5$, then $\sum_{r=1}^{12} f(r^2)$ is equal to :

(A) 216 **(B)** 219 **(C)** 221 **(D)** 223

157. The value of $\int_0^1 \frac{dx}{1+\sqrt{x}+\sqrt{1+x}}$ is $\frac{a}{b}-\frac{1}{\sqrt{b}}-\frac{\log(1+\sqrt{b})}{b}$ then $a-b$ is ________.

158. The value of the definite integral $\int_1^5 \sqrt{x-2\sqrt{x-1}}\,dx$ is ______________.

159. If $U_n=\int_0^{\pi}\frac{1-\cos nx}{1-\cos x}dx$, where n is positive integer or zero, then show that $U_{n+2}+U_n=2U_{n+1}$. Hence, deduce that $\int_0^{\pi/2}\frac{\sin^2 n\theta}{\sin^2\theta}=\frac{1}{2}n\pi$.

160. If $|a|<1$, show that $\int_0^{\pi}\frac{\log(1+a\cos x)}{\cos x}dx=\pi\sin^{-1}a$

161. Evaluate $\int_0^{\pi/2}\operatorname{cosec}\theta\tan^{-1}(c\sin\theta)d\theta$.

162. Evaluate $\int_0^{\pi/2}\sec\theta.\tan^{-1}(a\cos\theta)d\theta$.

163. Evaluate $\int_0^{\infty}\frac{\tan^{-1}ax-\tan^{-1}x}{x}dx$, where a is a parameter.

164. $\int_{-1}^{1/2}\frac{e^x(2-x^2)dx}{(1-x)\sqrt{1-x^2}}$ is equal to:

(A) $\frac{\sqrt{e}}{2}(\sqrt{3}+1)$ **(B)** $\frac{\sqrt{3e}}{2}$ **(C)** $\sqrt{3e}$ **(D)** $\sqrt{\frac{e}{3}}$

165. If $\int_0^1 e^{x^2}(x-\alpha)dx = 0$, then:

(A) $1<\alpha<2$ **(B)** $\alpha<0$ **(C)** $0<\alpha<1$ **(D)** $\alpha=0$

166. $\int_0^\infty \frac{dx}{\left[x+\sqrt{x^2+1}\right]^3}$ is equal to:

(A) $\frac{3}{8}$ **(B)** $\frac{1}{8}$ **(C)** $-\frac{3}{8}$ **(D)** None of these

167. If $I_1 = \int\limits_{-100}^{101} \frac{dx}{\left(5+2x-2x^2\right)\left(1+e^{2-4x}\right)}$ and $I_2 = \int\limits_{-100}^{101} \frac{dx}{5+2x-2x^2}$, then $\frac{I_1}{I_2}$ is:

(A) 2 **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** $-\frac{1}{2}$

168. The value of the integral $\int\limits_{-3\pi/4}^{5\pi/4} \frac{(\sin x+\cos x)}{e^{x-\pi/4}+1}dx$ is:

(A) 0 **(B)** 1 **(C)** 2 **(D)** None of these

169. $\int\limits_{2-a}^{2+a} f(x)dx$ is equal to [where $f(2-\alpha) = f(2+\alpha) \quad \forall \alpha \in R$]

(A) $2\int\limits_{2}^{2+a} f(x)dx$ **(B)** $2\int\limits_0^a f(x)dx$ **(C)** $2\int\limits_0^2 f(x)dx$ **(D)** None of these

170. $f(x) > 0 \ \forall x \in R$ and is bounded.

If $\lim\limits_{n\to\infty}\left[\int\limits_0^a \frac{f(x)dx}{f(x)+f(a-x)} + a\int\limits_a^{2a} \frac{f(x)dx}{f(x)+f(3a-x)} + a^2\int\limits_{2a}^{3a} \frac{f(x)dx}{f(x)+f(5a-x)} + \ldots\ldots\right.$

$\left.\ldots\ldots + a^{n-1}\int\limits_{(n-1)a}^{na} \frac{f(x)dx}{f(x)+f\left[(2n-1)a-x\right]}\right] = 7/5$ (where $a < 1$), then a is equal to:

(A) $\frac{2}{7}$ **(B)** $\frac{1}{7}$ **(C)** $\frac{14}{19}$ **(D)** $\frac{9}{14}$

171. If $f(x) = \int\limits_0^\pi \frac{t\sin t\, dt}{\sqrt{1+\tan^2 x\sin^2 t}}$ for $0 < x < \frac{\pi}{2}$, then:

(A) $f\left(0^+\right) = -\pi$

(B) $f\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8}$

(C) f is continuous and differential in $\left(0, \frac{\pi}{2}\right)$

(D) f is continuous but not differentiable in $\left(0, \frac{\pi}{2}\right)$

172. If $\displaystyle\int_{-\pi/4}^{3\pi/4} \frac{e^{\pi/4}dx}{\left(e^x + e^{\pi/4}\right)(\sin x + \cos x)} = k \int_{-\pi/2}^{\pi/2} \sec x dx$, then the value of k is:

(A) $\frac{1}{2}$ **(B)** $\frac{1}{\sqrt{2}}$ **(C)** $\frac{1}{2\sqrt{2}}$ **(D)** $-\frac{1}{\sqrt{2}}$

173. $\displaystyle\int_0^{\infty}\left(\frac{\pi}{1+\pi^2x^2} - \frac{1}{1+x^2}\right)\log x \, dx$ is equal to:

(A) $-\frac{\pi}{2}\ln \pi$ **(B)** 0 **(C)** $\frac{\pi}{2}\ln 2$ **(D)** None of these

174. If $A = \displaystyle\int_0^{\pi} \frac{\cos x}{(x+2)^2} dx$, then $\displaystyle\int_0^{\pi/2} \frac{\sin 2x}{x+1} dx$ is equal to:

(A) $\frac{1}{2} + \frac{1}{\pi+2} - A$ **(B)** $\frac{1}{\pi+2} - A$ **(C)** $1 + \frac{1}{\pi+2} - A$ **(D)** $A - \frac{1}{2} - \frac{1}{\pi+2}$

175. $\displaystyle\int_0^1 \frac{\tan^{-1}}{x} dx$ is equal to:

(A) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin x}{x} dx$ **(B)** $\displaystyle\int_0^{\frac{\pi}{2}} \frac{x}{\sin x} dx$ **(C)** $\displaystyle\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x}{x} dx$ **(D)** $\displaystyle\frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{x}{\sin x} dx$

176. The value of the definite integral $\displaystyle\int_0^{\pi/2} \frac{\sin 5x}{\sin x} dx$ is:

(A) 0 **(B)** $\frac{\pi}{2}$ **(C)** π **(D)** 2π

177. If $x = \displaystyle\int_c^{\sin t} \sin^{-1} z dz$, $y = \displaystyle\int_k^{\sqrt{t}} \frac{\sin z^2}{z} dz$, then $\frac{dy}{dx}$ is equal to:

(A) $\frac{\tan t}{2t}$ **(B)** $\frac{\tan t}{t^2}$ **(C)** $\frac{\tan t}{2t^2}$ **(D)** $\frac{\tan t^2}{2t^2}$

178. A function f is continuous for all x (and not everywhere zero) such that $f^2(x) = \displaystyle\int_0^x f(t) \frac{\cos t}{2+\sin t} dt$. Then $f(x)$ is:

(A) $\frac{1}{2}\ln\left(\frac{x+\cos x}{2}\right); x \neq 0$ **(B)** $\frac{1}{2}\ln\left(\frac{3}{2+\cos x}\right); x \neq 0$

(C) $\frac{1}{2}\ln\left(\frac{2+\sin x}{2}\right); x \neq n\pi, n \in I$ **(D)** $\frac{\cos x + \sin x}{2+\sin x}; x \neq n\pi + \frac{3\pi}{4}, n \in I$

179. If $A = \displaystyle\int_0^1 x^{50}(2-x)^{50} dx$, $B = \displaystyle\int_0^1 x^{50}(1-x)^{50} dx$, which of the following is true?

(A) $A = 2^{50}B$ **(B)** $A = 2^{-50}B$ **(C)** $A = 2^{100}B$ **(D)** $A = 2^{-100}B$

180. $\int_0^\infty \frac{\sin^2 x}{x^2} dx$ must be same as:

(A) $\int_0^\infty \frac{\sin x}{x} dx$ **(B)** $\left(\int_0^\infty \frac{\sin x}{x} dx\right)^2$ **(C)** $\int_0^\infty \frac{\cos^2 x}{x^2} dx$ **(D)** None of these

181. If $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$, then $\int_0^\infty \frac{\sin^3 x}{x} dx$ is equal to:

(A) $\pi / 2$ **(B)** $\pi / 4$ **(C)** $\pi / 6$ **(D)** $3\pi / 2$

182. $\int_0^x \frac{2^t}{2^{[t]}} dt$, where [•] denotes the greatest integer function, and $x \in R^+$, is equal to:

(A) $\frac{1}{\ln 2}\left(\left[x\right]+2^{\left\{x\right\}}-1\right)$ **(B)** $\frac{1}{\ln 2}\left(\left[x\right]+2^{\left\{x\right\}}\right)$

(C) $\frac{1}{\ln 2}\left(\left[x\right]-2^{\left\{x\right\}}\right)$ **(D)** $\frac{1}{\ln 2}\left(\left[x\right]+2^{\left\{x\right\}}+1\right)$

183. Let $f : R \to R$ be a continuous function and $f(x) = f(2x)$ is true $\forall x \in R$. If $f(1) = 3$, then the value of $\int_{-1}^{1} f\left(f(x)\right) dx$ is equal to:

(A) 6 **(B)** 0 **(C)** $3f(3)$ **(D)** $2f(0)$

184. $\int_{-1}^{2} \left[\frac{[x]}{1+x^2}\right] dx$, where [•] denotes the greatest integer function, is equal to:

(A) -2 **(B)** -1 **(C)** Zero **(D)** None of these

185. If the function $f : [0, 8] \to R$ is differentiable, then for some $0 < \alpha, \beta < 2$, $\int_0^8 f(t) dt$ is equal to:

(A) $3\left[\alpha^3 f\left(\alpha^2\right)+\beta^2 f\left(\beta^2\right)\right]$ **(B)** $3\left[\alpha^3 f(\alpha)+\beta^3 f(\beta)\right]$

(C) $3\left[\alpha^2 f\left(\alpha^3\right)+\beta^2 f\left(\beta^3\right)\right]$ **(D)** $3\left[\alpha^2 f\left(\alpha^2\right)+\beta^2 f\left(\beta^2\right)\right]$

186. Let $f(x)$ be positive, continuous, and differentiable on the interval (a, b) and $\lim_{x \to a^+} f(x) = 1$, $\lim_{x \to b^-} f(x) = 3^{1/4}$. If $f'(x) \ge f^3(x) + \frac{1}{f(x)}$, then the greatest value of $b - a$ is:

(A) $\frac{\pi}{48}$ **(B)** $\frac{\pi}{36}$ **(C)** $\frac{\pi}{24}$ **(D)** $\frac{\pi}{12}$

***187.** Let $f(x) = \int_1^x \frac{3^t}{1+t^2} dt$, where $x > 0$. Then :

(A) for $0 < \alpha < \beta$, $f(\alpha) < f(\beta)$ **(B)** for $0 < \alpha < \beta$, $f(\alpha) > f(\beta)$

(C) $f(x) + \pi / 4 < \tan^{-1} x \ \forall x \ge 1$ **(D)** $f(x) + \pi / 4 > \tan^{-1} x \ \forall x \ge 1$

***188.** Let $f:[1,\infty)\to R$ and $f(x)=x\int\limits_1^x \frac{e^t}{t}dt-e^x$. Then :

(A) $f(x)$ is an increasing function **(B)** $\lim\limits_{x\to\infty} f(x)\to\infty$

(C) $f'(x)$ has a maxima at $x=e$ **(D)** $f(x)$ is a decreasing function

189. Let f be a continuous function on $[a, b]$. Prove that there exists a number $x\in[a,b]$ such that $\int\limits_a^x f(t)=\int\limits_x^b f(t)dt$.

190. A continuous real function f satisfies $f(2x)=3f(x)\ \forall x\in R$. If $\int\limits_0^1 f(x)\,dx=1$, then find the value of $\int\limits_1^2 f(x)dx$.

191. Let $P(x)$ be a polynomial of least degree whose graph has three points of inflection $(-1,-1)$, (1, 1) and a point with absicissa 0 at which the curve is inclined to the axis of abscissa at an angle of $60°$. Then find the value of $\int\limits_0^1 P(x)\,dx$.

192. Let $f(x)=\int\limits_0^x |2t-3|dt$. Then discuss continuity and differentiability of $f(x)$ at $x=\frac{3}{2}$.

193. A periodic function with period 1 is integrable over any finite interval. Also, for two real numbers a, b and for two unequal non-zero positive integers m and n, $\int_a^{a+n} f(x)\,dx=\int_b^{b+m} f(x)\,dx$. Calculate the value of $\int_m^n f(x)\,dx$.

194. Prove that $y=\int_{1/8}^{\sin^2 x}\sin^{-1}\sqrt{t}\,dt+\int_{1/8}^{\cos^2 x}\cos^{-1}\sqrt{t}\,dt$, where $0\le x\le \pi/2$, is the equation of a straight line parallel to the x-axis. Find its equation.

195. Prove that $\int_0^1\sqrt{(1+x)\left(1+x^3\right)}\,dx$ cannot exceed $\sqrt{15/8}$.

196. Let $A=\int\limits_0^\infty \frac{\log x}{1+x^3}dx$. Then find the value of $\int\limits_0^\infty \frac{x\log x}{1+x^3}dx$ in terms of A.

197. Let $f(n)=1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}$. Then show that $f(n)=\int\limits_0^{\pi/2}\cot\left(\frac{\theta}{2}\right)\left(1-\cos^n\theta\right)d\theta$.

198. If $f(x)=\frac{\sin x}{x}\ \forall\ x\in(0,\pi]$, prove that $\frac{\pi}{2}\int\limits_0^{\pi/2} f(x)f\left(\frac{\pi}{2}-x\right)dx=\int\limits_0^\pi f(x)dx$.

199. Let $\int_{x}^{x+p} f(t)dt$ be independent of x and $I_1 = \int_0^p f(t)dt$, $I_2 = \int_{10}^{p^n+10} f(z)dz$ for some p, where $p, n \in N$. Then evaluate $\frac{I_2}{I_1}$.

200. Suppose f is a real-valued differentiable function defined on $[1, \infty)$ with $f(1) = 1$. Moreover, support that f satisfies $f'(x) = \frac{1}{x^2 + f^2(x)}$. Show that $f(x) < 1 + \frac{\pi}{4} \forall x \geq 1$.

201. Let f be a continuous function on [a, b]. If $F(x) = \left(\int_a^x f(t)\,dt - \int_x^b f(t)\,dt\right)\left(2x - (a+b)\right)$, then prove that there exist some $c \in (a,b)$ such that $\int_a^c f(t)\,dt - \int_c^b f(t)dt = f(c)\,(a + b - 2c)$.

202. If $f(x) = x + \int_0^1 t(x+t)\, f(t)\,dt$, then find the value of the definite integral $\int_0^1 f(x)dx$.

203. Consider a real-valued continuous function f such that $f(x) = \sin x + \int_{-\pi/2}^{\pi/2} (\sin x + tf(t))dt$. If M and m are maximum and minimum values of the function f, then the value of M/m is:

204. If the value of the definite integral $\int_0^1 {}^{207}C_7 x^{200} \cdot (1-x)^7\, dx$ is equal to $\frac{1}{k}$, where $k \in N$, then the value of $k/26$ is:

205. If the value of the definite integral $\int_0^1 \frac{\sin^{-1}\sqrt{x}}{x^2 - x + 1}\, dx$ is $\frac{\pi^2}{\sqrt{n}}$ (where $n \in N$), then the value of $n/27$ is: